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Developmental Processes and Stages in the Acquisition of Cardinality

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This is a study of the level of children's understanding of cardinality, focusing on the difference between a true cardinality response and the application of a mechanically learned rule. The authors also evaluate and discuss the possible relationship between cardinality and counting. The subjects were two groups of 32 preschool children, ranging in age from 4 years 3 months to 6 years 3 months. Experimental methodology included two large sets of tests (elements–cardinal vs cardinal–elements), using both numbers and vowels with forward vs backward counting, and visual vs verbal presentation conditions. Results show that cardinality responses are affected by both the direction and nature of the elements in the counting sequence. Scrutiny of errors committed in the various tests enables us to suggest six stages in the acquisition of cardinality. Although there appears to be a developmental dependency between counting and cardinality, this relationship is not significant in all cases.

INTRODUCTION

Developmental Processes and Stages in the Acquisition of Cardinality

A review of current literature on the cardinal meaning of numbers enables one to observe two main lines of resarch. In the first, children indicate the number of objects within a set (Fuson, Pergament, Lyons, & Hall, 1985b; Gelman & Gallistel, 1978; Schaeffer, Eggleston, & Scott, 1974; Wilkinson, 1984, etc.). In the second, children establish the relationship of equivalence or inequality between two different sets. In turn there are different

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approaches: (a) one which is basically concerned with the role of the one-to-one correspondence (Brainerd, 1979; Kingma & Koops, 1984; Michie, 1985; Piaget & Szeminska, 1941), and (b) another which stresses the role of counting (Clements, 1984; Gelman, 1982; Markman, 1979; Michie, 1984; Saxe, 1979).

In the former line of research, specification of the cardinal usually presupposes the use of counting. Work carried out by Gelman and Gallistel (1978) showed that acquisition of cardinality occurred after the acquisition of one-to-one correspondence and stable order. Along this same line of research, Wilkinson (1984) suggested that counting and cardinality are closely linked to each other during the early and advanced phases of counting skills development, but that they may become dissociated during the intermediate period. Moreover, with regard to Gelman and Gallistel's (1978) position, Wilkinson pointed out that elementary counting skills may be acquired even earlier than cardinality, but that cardinality reaches functional maturity prior to counting.

In the second line of research, when two sets are compared the child may either carry out an item-to-item correspondence between the elements of both sets, or obtain the cardinal value of each, in order to then compare them. From the first approach (a), Piaget and Szeminska (1941), according to their logical model, suggested that it is the synthesis between class and asymmetrical relationships, not verbal enumeration, that leads to the operational conservation of the number. In contrast, other authors such as Clements (1984), Fuson and Hall (1983), Fuson et al. (1985b), Gelman and Gallistel (1978), and Saxe (1979), hypothesised that the development of numerical concepts and skills is derived from the integration of counting, subitising, and estimation skills. Michie (1984) integrated these two divergent theoretical conceptions, concluding that the absolute number ("How many?") appears to develop before the relative concept ("Which has more?"). Michie argued that those children who used counting to determine the cardinal or absolute value of a set were reluctant to use the same procedure in certain relational situations. On the other hand, Brainerd (1979) suggested that the development of the concept of number is rooted in ordination, and that ordination is an indispensable prerequisite for the child to truly acquire cardination. Michie (1985), however, concerned with the developmental sequence of the numerical skills of cardination and order, questioned Brainerd's theory, claiming that children understand number as an absolute quantity before they can understand ordered series.

With regard to the latter approach of this line of research, (b), Markman (1979) presented results in which the use of collection terms better facilitated the understanding of cardinality than did class terms. However, while some studies have supported Markman's position on concepts such as class-inclusion (Bermejo, 1989; Fuson, Lyons, Pergament, Hall, & Kwon, 1988), her findings have not been confirmed vis-a-vis cardinality (e.g. Fuson, Pergament, & Lyons, 1985a; Hodges & French, 1988). Within this same orientation, Saxe (1979) has described both a quantitative and a prequantitative method of determining equivalence or inequality between two sets. Children demonstrate a quantitative approach when they use counting to judge the relationship between sets, and a prequantitative approach when they use procedures other than counting in making their estimation. Similarly, Michie (1984) found that counting is more efficient when the elements of the two sets are placed in separate boxes after they have been counted, than when they are placed in two rows which the children can see.

To conclude, Gelman and Gallistel (1978) have argued that cardinality depends on the ability to count, thus highlighting the close relationship between these two concepts. Schaeffer et al. (1974) proposed that children acquire cardinality via an integration of the earlier processes of subitising and counting. Wilkinson (1984) found that counting and cardinality are strongly associated to one another during the early and late development of counting but that they are dissociated during the intermediate period. Fuson and Hall (1983) have suggested two possible levels of comprehension when a child's response to "How many?" is the last counting word. The first level of comprehension could be a mechanically learned reaction, which is not indicative of cardinality. A second and more advanced level of comprehension indicates the child's reference to the entire set of objects, and is considered to be a response of true cardinality. The authors added that all children do not necessarily pass through the first level of comprehension (Fuson, 1988).

This brief review reveals the absence of consistent and definitive data about the process of cardinality acquisition. This paper attempts to study the cognitive processes that children follow in acquiring cardinality. Specifically we focus not only on the steps children follow toward achieving cardinality, but also on the possible relationships between counting, the rule of "How many", and the principle of cardinality. The rule manifests itself when the child, faced with the question "How many ___ are there?", after counting merely and exclusively repeats the last word or term in the sequence given. Cardinality, on the other hand, means that the child's response refers to the numerosity of the whole set of elements presented, though is at times not necessarily the last symbol pronounced as in backwards counting tasks.

We suppose that there is a certain "cultural" relationship between counting and cardinality, but not necessarily a theoretical relationship. The cultural relationship would be due to the fact that the learning of cardinality is normally associated with the teaching of counting, which comes earlier than cardinality, as Fuson (1988), Gelman and Gallistel (1978), Kingma and Koops (1984), Schaeffer et al. (1974) and, partially, Wilkinson (1984) have argued. However, counting and cardinality could very well be two independent abilities, given that children can count (perhaps perfectly well) without cardinality, and vice-versa (Fuson, Pergament, Lyons, & Hall, 1985b; Russac, 1983). Furthermore, the cardinal number can be determined not only by means of counting but also by other quantificators such as: subitising, and estimation (Klahr & Wallace, 1976).

In the current investigation we analyse various types of counting and cardinality behaviours. We especially focus on children's mistakes in the following experimental situations: familiar vs novel tasks, counting forwards vs counting backwards, sequences of number words vs sequences of vowels, and elements–cardinal situation vs cardinal–elements situation (see Fig. 2). We hypothesise that the children's responses in the familiar situations will be more or less influenced by automatised mechanisms, which are very difficult to analyse; the novel situations, on the other hand, will limit the influence of these mechanisms, and thus will facilitate both the manifestation of the cognitive processes that underlie the acquisition of cardinality, as well as our inferences and understanding of those processes. In first block of tests (i.e. Elements–Cardinal) the child determines the cardinality of a given set of elements, while in the second block of tests (i.e. Cardinal–Elements) the process is reversed, and the child determines the elements pertaining to a given cardinal. We expect that these complementary situations will differentiate the various levels of comprehension in cardinality. Finally, the counting backwards situation allows us to differentiate empirically the "How many" rule from the principle of cardinality.

METHOD

Subjects

The subjects in this study were 64 students in the first and second year of a public preschool in Madrid. They came from middle-class backgrounds and were chosen at random. Group I consisted of 32 children between the ages 4 years 3 months and 5 years 2 months (M = 4 years 7 months). Group II consisted of the remaining 32 children, whose ages varied between 5 years 4 months and 6 years 3 months (M = 5 years 7 months). Each group was composed of equal numbers of boys and girls.

Materials

We used up to a maximum of six red chips, each measuring 1 cm in diameter, to conduct the experimental tasks described below in the empirical procedure section. Secondly, we employed two white cards

(a)	(b)	(c)
2 ○ ○	○ ○ ○ ○	a e i o u 1 2 3 4 5

(a) Backwards counting practice card; (b) vowel-counting practice card; and (c) card indicating vowel–numeral correspondence during vowel tasks.

FIG. 1. Empirical material presented during the experimental session.

Elements–Cardinal				*Cardinal–Elements*			
Numbers		*Vowels*		*Numbers*		*Vowels*	
Counting forwards (2 & 5)	Counting backwards (3 & 4)	Counting forwards (2 & 5)	Counting forwards (3 & 4)	Verbal (2 & 5)	Visual (3 & 4)	Verbal (2 & 5)	Visual (3 & 4)

FIG. 2. Table of the empirical design.

(14 × 20 cm) on which were pasted two and three red circles, respectively (1.5 cm each), as well as the corresponding numeral "2" or "3" just above the last element in the row. These two cards were used during a phase of the study in which children practiced the backwards counting sequence beginning with these numerals (see Fig. 1.a). Another white card (14 × 20 cm), fashioned after the cards already described, had four red circles in a row (1.5 cm each), and was used for vowel-counting practice (see Fig. 1.b). An additional white card (14 × 20 cm) contained a row of vowels pasted above a corresponding row of numerals (see Fig. 1.c). Finally, we employed four cards (7 × 10 cm) each of which contained either a numeral or a vowel printed in lower case to request the child to hand in the number of chips indicated (see Empirical Procedure). In order to facilitate the children's approach to the task and make them feel they were in a familiar situation, the experimenter introduced a puppet "Espinete" (a well-known TV puppet in Spain).

Empirical Procedure

Each child took all the tests individually during school hours, in research sessions that lasted approximately 20 minutes each. The empirical procedure consisted of presenting two blocks of tasks (the mode of presentation factor) in such a way that the first (elements–cardinal) began by asking the child to count the elements of one set and to indicate afterwards the cardinal number of the set; while in the second (cardinal–elements) children were presented a cardinal, and were asked to determine, from a row of six objects, the elements which corresponded to the mentioned cardinal (see Fig. 2). Each block of tasks consisted of two types of tests (counting sequence factor), such that some trials used numbers and others used the standard vowel sequence for counting. In the first block, using number words, the children carried out two tasks: one counting forwards and another counting backwards (counting direction factor), both followed by the question: "How many chips are there?". The counting backwards situation allowed us to, above all, differentiate between cardinality and the "How many" rule. In both cases we presented two successive sets of chips in a horizontal line consisting of 2 and 5 chips in the forwards counting condition ("Go ahead and count these chips"), and 3 and 4 in the backwards counting condition. In this latter case the children were asked to begin counting backwards where the starting word was one more than the cardinality of the set to be counted ("Go ahead and count these chips backwards, starting from 4 [or 5]"). In the vowel condition, we presented four tests with 2, 3, 4 and 5 chips in a row, and the children were asked to count forwards. We did not include a backwards task, due to the difficulty of counting backwards with vowels at these ages. The children were requested: "Go ahead and count the chips using vowels", and were subsequently asked "How many chips are there?" The child was requested to respond to these questions with vowels. In this case, as in all the vowel tasks, the children could make use of the vowel–numeral correspondence card that was placed in front of them (see Fig. 1.c). Before introducing the vowel-counting task, however, we checked the subjects' ability to produce the standard vowel sequence, and provided a brief training to those who needed it, such that all could produce the sequence without problems before proceeding. Likewise, before starting the counting backwards test, the experimenter made sure the children understood each task, asking them to count backwards in one or two simple practice situations.

In the second block of tests (cardinal–elements) we asked the children for a precise set of objects, either using numbers or using vowels. In both cases, the request was made either verbally ("Go ahead and give Espinete 2 [or *e*] chips"), or visually by means of showing a card which had a vowel or a number written on it ("Go ahead and give Espinete these chips") (request form factor). The inclusion of both a verbal and a visual request factor will allow us more effectively to discriminate levels of cardinality acquisition. Our pilot research has pointed our attention to the differences in children's responses when presented with verbal vs visual requests. Furthermore, the visual presentation factor is intended to allow us to judge whether children use the visual cardinal as either a symbol of the entire set of objects, or merely as an indicator of the one object to which it is assigned. In the verbal test we asked for 2 and 5 chips, while in the visual test we asked for 4 and 3. The child responded by selecting the quantity requested by the experimenter from a line of 6 chips in front of him. There was only one trial in each experimental situation.

The order of presentation of the two blocks of tests was counter-balanced, as was that of the tests within each block. The subjects were assigned at random to each of the resulting orders. The order of presentation of the different trials within each test was also initially determined at random and was constant for all the subjects. However, in the counting backwards condition, the set with 3 chips was presented before the set of 4 chips in half the subjects, whilst the other half took the tests in the reverse order. Subjects' responses were dichotomised as either correct or incorrect. In the counting forwards condition cardinality response was rated as correct when the child repeated the last number word or the last counting vowel, while in the counting backwards condition, the only correct response was one that gave the exact cardinal value of the set.

ANALYSIS AND DISCUSSION

We have analysed our data both quantitatively and qualitatively. To this end we first carried out an arc–sin transformation of the proportions of correct trials. Using the BMDP2V program, the ANOVA 2 (Group I vs Group II) × 2 (Elements–Cardinal vs Cardinal–Elements) × 2 (Numbers vs Vowels) with repeated measures, as may be seen in Table 1, showed significant main effects for age (F [1,62] = 20.39, $P < 0.01$) and counting sequence (F [1,62] = 40.64, $P < 0.01$), indicating that the older children obtained better results than the younger children, and that the numerical tasks were, globally, easier than the vowels tasks. Likewise, the interaction

TABLE 1
Average and Standard Deviations from Transformed Proportions of Correct Responses in Cardinality

	Elements–Cardinal		*Cardinal–Elements*	
	Numbers	*Vowels*	*Numbers*	*Vowels*
Group I	1.78 (0.35)	1.35 (0.98)	2.33 (0.52)	1.15 (0.88)
Group II	2.08 (0.41)	2.20 (0.81)	2.53 (0.31)	1.70 (1.03)

Maximum possible score is 2.64.

of these two factors (F [1,62] = 6.08, $P < 0.05$) was significant as well as the interaction of presentation mode with counting sequence (F [1,62] = 30.89, $P < 0.01$). That is to say that when vowels were used as components of the counting sequence, there was a greater difference between the averages of Groups I and II than when numbers were used. However, the difference in averages between the elements–cardinal and cardinal–elements situations was more pronounced when the children worked with numbers than with vowels, although their performance was greater when faced with numbers than with vowels. The other results were not significant. Thus, there is no significant difference between both blocks of tasks (i.e. Elements–Cardinal and Cardinal–Elements). If in the general analysis we omit counting backwards and visual presentation, we obtain the same results in the ANOVA with respect to the significance of the factors and their interactions. We shall analyse the preceding data in greater detail below, differentiating the two main blocks of tests in order to make our exposition clearer.

Elements–Cardinal Block

With regard to the first block, the overall results with respect to cardinality showed that the children in Group II obtained a higher percentage of correct trials than those in Group I, except when they had to count forwards with numbers, in which case their success was the same. First we shall examine the results corresponding to the numerical tasks.

Numerical Tests. The ANOVA 2 (Group I vs Group II) × 2 (Counting Forwards vs Counting Backwards) showed significant main effects for age (F [1,62] = 10.77, $P < 0.01$) and counting direction (F [1,62] = 128.44, $P < 0.01$), as well as the interaction between them (F [1,62] = 7.82, $P < 0.01$). Thus the task of cardinality is more difficult with a backwards counting sequence, even though we used sets of only 3 or 4 objects (see Table 2). A minimal increase in the number of objects presented to the child (3 vs 4) decreases the success rates of all the subjects, but particularly in the younger group (Group I). The fact that children's counting success rates are very similar when counting forwards (100%), or when counting backwards with sets of 3 (68%) and 4 (62%) objects alike, strongly suggests that subitising is the main mechanism responsible for the correct cardinal response during backwards counting. Subitising in turn appears to be much easier with 3 than with 4 objects, particularly for the younger subjects (see Fuson, 1988). There are at least two phenomena that might explain the differences we have found between the two conditions (counting forwards and counting backwards). The first is that the children were generally more familiar with the counting forwards situation. The second

TABLE 2
Percentages of Correct Trials for the Cardinality and Counting Responses in the Elements–Cardinal Block With Numbers

	Counting forwards		Counting backwards	
	Counting	*Cardinality*	*Counting*	*Cardinality*
Group I	100	97	48	23
Group II	100	97	83	52

possibility is that the counting backwards technique permits us to discriminate between levels of a child's understanding of cardinality. So, while counting backwards allows us to distinguish empirically the "how many rule" and the "principle of cardinality", it does not effectively assess differences in standard counting. This limitation could produce a situation in which underlying cognitive operations produce responses considered as wrong answers in the counting backwards task but produce correct answers in the forward task, leading to incorrect inferences of cardinal understanding when only the forward task is given. This misunderstanding is frequent in studies of cardinality (see Gelman, Meck, & Merking, 1986; Wagner & Walters, 1982), and attempts to avoid this problem through the use of language aspects (see Fuson, 1988) probably inordinately complicate the experimental situation.

We have classified the subjects' mistakes into five types: (1) answering with the last number word of the sequence used; (2) answering with the first number word of the sequence; (3) counting again; (4) repeating the sequence used in the counting; and (5) responding with a random number (see Table 3). A great deal of the wrong answers of the older children and a

TABLE 3
Percentages of Cardinality Responses Given in the Backwards Counting Task

	Three objects		Four objects	
	Group I	*Group II*	*Group I*	*Group II*
Cardinal	41	72	6	31
Last number word	41	22	50	19
First number word	6	3	34	47
Counting again	6	3	6	3
Repeating the sequence	3	—	3	—
Random number word	3	—	—	—

considerable percentage of those of the younger ones consist of repeating the number word at which counting had begun. This behaviour may be due either to the fact that the child knew that the first number word used was the largest, and included all the numbers which followed it—although they did not notice that it did not really end with "1"—or to a certain understanding of the effects or meaning of the backwards sequence. A relative knowledge of the cardinal meaning of the numbers produced during the backwards counting is reflected in both cases. A third and possibly more plausible interpretation suggests that this sort of error may be produced by the effect of at least two factors: the difficulty of subitising four objects, and the salience (the first and the largest) of the first number word of the sequence employed. This accounts for the fact that almost all the children who make this error with four objects are successful when presented with three objects.

Children who made the first type of error behaved as if it were a standard count, directly using the rule of giving the last number word of the sequence used. This behaviour, typical in the younger children above all, is far from a perfect understanding of the principle of cardinality, although it is normally confused with the correct response of cardinality in situations of standard counting. Perhaps the children who committed type 3 and 4 errors (and, of course, type 5) are even further away from reaching the concept of cardinality.

Therefore, based on our observations of the children's behaviour in the backwards counting situation, we suggest the following stages (although not in a classic Piagetian sense) or steps in the acquisition of cardinality: (1) misunderstanding of the task and responding at random; (2) mere repetition of the previous counting sequence; (3) counting the objects again; (4) giving the final number word of the sequence used (the rule of "How many?"); (5) suggesting the largest numeral of the counting sequence; and (6) the response of cardinality. We believe that these are the stages that children in Western cultures normally follow towards the acquisition of cardinality in the standard situation, although not every child will necessarily pass through each and every stage. In the second stage the child does not make reference to the objects, while in the third stage a number–object correspondence is established. The fifth stage builds upon the fourth stage, with the idea of "largest" number used in the counting sequence. In other words, a child in the fourth stage responds with the final number in the given sequence, while the typical response in the fifth stage is to respond with the largest number in the given sequence. This developmental sequence which we propose for the acquisition of cardinality is rather different from that of Gelman and Gallistel (1978), but shares much in common with Fuson's (1988) position. Our sequence differs from Fuson's in that we have found a fifth stage that she did not consider, given the experimental situation she employed. Additionally we have more clearly delimited the fourth and sixth stages of the "How many" rule and cardinality proper, respectively.

As for counting in these same tasks, it was observed that both groups attained 100% of correct trials in the forwards counting condition. Backwards counting percentages were noticeably lower, which is probably due to the fact that backwards counting sequences are at times acquired as a new sequence of numerals (Fuson, Richards, & Briars, 1982) (see Table 2). All errors in the backwards counting task were sequence errors and can be classified into three main types: (1) the omission of a number word in the decreasing sequence (Group I 19% vs Group II 13%); (2) the use of a mixed sequence with both increasing and decreasing numbers (Group I 33% vs Group II 5%); (3) an increasing sequence (Group I 2%).

It is important to note the possible relationship between the counting skill, operationalised as the correct sequence of object–number word correspondences, and the response of cardinality in the situation of counting backwards. McNemar's test shows that there are no significant differences in any of the groups when the sample consists of three elements, but the differences are significant when the array consists of four elements (χ^2 $[1, n = 32] = 10.89$, $P < 0.01$ for both groups of subjects). It therefore seems clear in this latter case that the child may have acquired the counting skill before understanding cardinality, as Gelman and Gallistel (1978) and Schaeffer et al. (1974) have pointed out; but it could also be understood in

TABLE 4
Frequency Tables for the Counting Backwards Task

			Three objects						Four objects		
			Cardinality						Cardinality		
			C	I	T				C	I	T
Group I	Counting	I	5	12	17	Counting	I		2	14	16
		C	8	7	15		C		0	16	16
		T	13	19	32		T		2	30	32
			Cardinality						Cardinality		
			C	I	T				C	I	T
Group II	Counting	I	2	1	3	Counting	I		2	6	8
		C	21	8	29		C		8	16	24
		T	23	9	32		T		10	22	32

C, correct; I, incorrect; T, totals.

the first case (3 elements) that these are two distinct phenomena. In fact, counting successes are practically the same in the 3 and 4 objects conditions, especially among the younger children (see Table 4). However, the children's success changes significantly with respect to the cardinality response with 3 and 4 objects. This again suggests the importance of subitising to respond correctly to cardinality questions, as can be inferred from the fact that some children count incorrectly and yet still correctly answer the cardinality question (see also Russac, 1983). How could this be if counting is an essential component of cardinality? Might it not be more appropriate to speak of counting merely as one of the quantification procedures (see Klahr & Wallace, 1976) that can be used to specify the cardinal of a set, or even the sum of two sets? (see Bermejo & Rodríguez, 1987). It is for this reason that we suggested in the Introduction the existence of a cultural or situational, but not necessarily theoretical, relationship between counting and cardinality.

Tasks With Vowels. As for the influence of the elements that made up the counting sequence, the use of vowels instead of numbers significantly reduced the percentage of correct cardinality trials in the forwards counting task. We found a significant difference between counting forwards with numbers and vowels (with 2 and 5 objects) (F [1,62] = 44.62, $P < 0.01$), as well as a significant group difference (F [1,62] = 8.53, $P < 0.01$). These results cannot be attributed to ignorance of the vowels on the part of the younger children, since our study procedure ensured a uniform ability to recite the vowels without difficulty. In addition, we know that even very young children can discriminate numbers and letters, although they may not know the structural and functional differences between them.

Regarding cardinality in the vowel condition, the ANOVA 2 (Group I vs Group II) × 2 (2 and 5 vs 3 and 4 objects) showed significant main effects for age (F [1,62] = 13.17, $P < 0.01$) and for set size (F [1,62] = 4.2, $P < 0.05$) (see Table 5). Furthermore, the most frequently observed mistakes consisted of counting again and repeating the sequence of vowels employed in the counting, which we have described as stages 3 and 2 (see Table 6). Therefore, the introduction of vowels seems to bring about a return to patterns of behaviour that have already been overcome with regard to numbers (e.g. Markman, 1979; Saxe, Gearhart, & Guberman, 1984; Schaeffer et al., 1974). These children probably consider the vowels as mere labels (Sinclair & Sinclair, 1984), without granting them the cardinal meaning of the numbers. Besides, even when the task entails greater complexity than the standard task with numbers, their errors do not seem to be attributable to deficiencies in coordination or memory, for the percentage of correct counting is higher than the percentage of correct answers of cardinality, and part of the wrong trials are due to the entire repetition of the vowel sequence used in the counting process, particularly

TABLE 5
Percentages of Correct Trials for the Cardinality and Counting Responses in the Elements–Cardinal Block With Vowels

	Two and five objects		*Three and four objects*	
	Counting	*Cardinality*	*Counting*	*Cardinality*
Group I	72	42	72	38
Group II	100	80	98	80

in Group I. Consequently, these are all signs that there may be a lack of attribution of meaning. The same may be said in regard to the response of counting again, for, as Fuson and Hall (1983) have pointed out, it could be a way of specifying all objects, such that the child's response refers to each of the elements in the set, but not of the total or cardinal.

We also found markedly higher performance on counting trials than on cardinality trials (see Table 5). This high success rate seems to ratify our assertion that we are faced with the "assignment of cardinal meaning" problem discussed above. As in the counting backwards test, the errors were mainly ones of sequence, and were only found in the group of younger children. We can categorise these errors into four types: (1) they omit one vowel or repeat a vowel; (2) they alter the order of the sequence; (3) they omit a chip or count a chip twice; and (4) they substitute the vowels for numbers (see Table 6).

Regarding the relationship between counting with vowels and success in cardinality in Group I (see Table 7), McNemar's test is significant

TABLE 6
Percentages of Cardinality and Counting Errors With Vowels Within the Elements–Cardinal Block

	Group I	*Group II*
Cardinality errors		
Counting vowels again	39	35
Repeating the sequence	31	8
Sequence of number words	16	—
Random vowel	14	7
Counting errors		
Omit vowels of the correct sequence	61	—
Repeating a vowel	11	—
Alter the order of the sequence	11	—
Omit a chip	6	—
Double-counting of a chip	6	—
Substitute the vowels for number words	5	—

TABLE 7
Frequency Tables for the Task of Counting With Vowels

		Group I					*Group II*		
		Cardinality					Cardinality		
		C	I	T			C	I	T
Counting	I	4	6	10	Counting	I	0	0	0
	C	8	14	22		C	25	7	32
	T	12	20	32		T	25	7	32

C, correct; I, incorrect; T, totals.

(χ^2 [1, n = 32] = 5.55, $P < 0.05$), both globally, when three or more correct trials out of four, was considered a correct response, as well as when analysed individually for every set of objects (i.e. for 2, 3, 4, and 5 objects respectively). The children in Group II routinely count correctly, but some respond incorrectly in the cardinality task (see Table 7). Our findings are similar to and partially support the findings of Gelman and Gallistel (1978) and Wilkinson (1984) in that we observed that counting is acquired prior to cardinality, independent of the number of objects or age of subjects. However, some children count incorrectly but respond correctly to the cardinality tasks, sustaining the results found in our counting backwards situation. Counting appears to be a quantification procedure closely related to cardinality in certain contexts. However, counting is probably not an essential component of cardinality, despite the position of Schaeffer et al. (1974).

Cardinal–Elements Block

To avoid repeating ourselves we will present a brief summary of our findings in this section. We analysed our data with a repeated measures ANOVA 2 (Group I vs Group II) × 2 (Numbers vs Vowels) × 2 (Visual vs Verbal), as shown in Table 8. We found significant main effects for the three factors ($F[1,62] = 7.18$, $P < 0.01$; $F[1,62] = 74.37$, $P < 0.01$ and $F[1,62] = 21.57$, $P < 0.01$ in this order), and a significant interaction of age with request form ($F[1,62] = 5.39$, $P < 0.05$). Once again we observe that vowels imply a complication of the task and that Group II carried out their task more successfully than Group I. Additionally we find tasks presented visually are more difficult than those presented verbally, and that the difference of averages between both groups was greater when the cardinal was presented visually. We will now examine these findings in greater detail.

TABLE 8
Averages and Standard Deviations from Transformed Proportions of Correct Responses in Cardinality Within the Cardinal–Elements Block

	Numbers		*Vowels*	
	Verbal	*Visual*	*Verbal*	*Visual*
Group I	2.36 (0.20)	1.99 (0.71)	1.35 (0.80)	1.09 (0.67)
Group II	2.41 (0.00)	2.25 (0.50)	1.70 (0.83)	1.65 (0.84)

Maximum possible score is 2.41.

Numerical Tasks. We observed that the subjects of both groups carry out the verbally requested tasks more effectively than the visually requested tasks (see Table 8). Likewise, within the visual situation, one can appreciate two patterns of erroneous behaviour that did not arise when the cardinal was presented verbally: (1) randomly giving in any or all chips; and (2) counting the chips correctly and giving in only the chip to which the number corresponding to the cardinal in question is assigned (i.e. giving the fourth chip, when asked to give four chips). The first of these errors appeared in the group of younger children (13% of trials) and corresponds to the aforementioned first stage because this behaviour involves, at least partially, a miscomprehension of the situation and a random choice of the number of elements given. The second type of error was made with approximately equal frequency in both groups (9% and 6% of trials in Groups I and II respectively) and would be typical of the fourth stage. In committing this second type of error, the child focuses on the last number word of the counting sequence, as he does in following the "How many" rule, but additionally is able to understand that the last number word repeated represents a particular (the last-counted) object. This response is more frequent in the tests with vowels than in tests with numbers (see Table 9 for vowel task results). It would be quite interesting to analyse how that same number word becomes a label of the whole set in a latter developmental moment, though our current data does not allow us to make this analysis.

Tasks With Vowels. When the cardinal was requested by means of a vowel, we found that on the one hand the number of correct trials was lower, and on the other carrying out of tests was worse with visual than with verbal presentation, just as occurs in the former (numeral presentation) situation (see Table 8). With regard to the drop in the children's success when using vowels, it seems reasonable to suggest that this was due to a difficulty in attributing quantitative or cardinal meaning to the vowels presented, as we point out previously, and as may be seen in the high number of errors based on random choices. It may also be that a child's correct answer in this situation implies either his ability to count with vowels, or his ability to carrying out correspondences between vowels and numbers. In both cases the cognitive processes involved in problem solving would be longer and more complicated in this test than in the one in which only numbers are used. As for the significance of the request form, it could be conjectured that it is more difficult to identify the meaning of the written symbol, whether this be a numeral or a vowel, than to understand this same symbol when it is expressed verbally, given that the level of children's language development at these ages (particularly in Group I) is fundamentally verbal and not written.

There were also differences in the pattern of errors made with vowels between the two groups (see Table 9). The older subjects basically made two types of errors: (a) counting well with vowels, but handing in only the chip to which the vowel suggested as a cardinal is assigned, instead of the group of chips requested, which is typical of the fourth stage; and (b) giving in all the chips without counting, which is typical of behaviour in the first stage of cardinality development. On the contrary, we observed five types of errors in the younger sample: the two already mentioned; (c) handing in a random number of chips; (d) randomly handing in any single chip; and finally (e) counting all the chips present at the outset and then handing all

TABLE 9
Percentages of Cardinality Responses in the Cardinal–Elements Block With Vowels

	Verbal		*Visual*	
	Group I	*Group II*	*Group I*	*Group II*
Correct answers	38	58	23	55
Counting well and giving in an extra chip	—	2	3	2
Counting well and giving in only the chip corresponding to the last counting word	13	31	13	31
Counting once and giving in all the chips	6	—	6	—
Giving in all chips without counting	18	9	20	12
Giving in different chips at random	16	—	16	—
Giving in a single chip at random	9	—	19	—

of them in. The last error mentioned corresponds to the third stage, while the two former errors fall under the first stage.

CONCLUSIONS

Our analyses show that the older children better understood the notion of cardinality than did the younger ones, and that these age differences were consistent over every test we included in this study. Furthermore, differences between the two groups increased as the complexity of the tasks increased. Likewise, our results allow us to conclude that familiar situations (counting forwards with numbers) are simpler than novel situations (counting backwards and using vowels). In the former case the children's success rate was clearly high, whilst in the latter it dropped globally to under 25%. This "novel situation" research methodology has enabled us to avoid the traditional difficulties inherent in research with young children (2–3 years) and successfully work with preschool subjects to measure the development of cardinality, even as it is just unfolding. These new tests not only enable us to specify the levels of cardinality acquisition, but also to analyse those cognitive processes that intervene in the acquisition of cardinality. More specifically the situation of counting backwards, for example, empirically demonstrates the existence of a developmental stage (the rule of "how many") prior to cardinality which had been difficult to investigate with previous counting methodology.

Some authors underestimate the possible relationship between counting and cardinality (see, e.g. Piaget & Szeminska, 1941); while others insist on the importance of this relationship (see, e.g. Gelman & Gallistel, 1978). Piaget and Szeminska maintain that counting is acquired after cardinality, and on the contrary Gelman and Gallistel claim that counting is developed before cardinality. Our data from both number and vowel tasks suggest that counting is acquired before cardinality. This significant relationship between counting and cardinality may be due in part to how we empirically measured these phenomena, or to how Western cultures usually use counting to determine a cardinal exactly. Our findings of counting before cardinality does not necessarily rule out a cardinality without counting that is arrived at by subitising. We see this particularly clearly in the performance of children who respond incorrectly in terms of counting, but correctly in terms of cardinality (see also Russac, 1983). Our research (Bermejo, Lago, & Rodríguez, 1989) confirms that in specific situations counting is a quantification procedure closely related to, but not an essential component of, cardinality, as Schaeffer et al. (1974) have suggested.

Error analysis has fundamentally led us to suggest the existence of six stages (though not in a strong Piagetian sense) in the acquisition of cardinality in the standard counting situation: (1) incomprehension of the situation and random response; (2) repetition of the number word sequence given in the counting; (3) counting the objects again; (4) giving the last number word of the sequence used (the rule of "how many"); (5) responding with the largest number word of the given sequence; and (6) a true cardinality response. The second and third stages are differentiated basically in that in the former the child does not refer to objects, while in the latter he carries out a strict number word–object correspondence. The fourth stage is an important step towards cardinality, for the child not only knows that each number word in the series represents an object in the set, which is typical of the third stage, but also can correctly answer the question "how many are there?" by giving the last counting word. In the fifth stage, the child knows that the cardinal corresponds to the largest number word of the given sequence. However, it is not until the following (sixth) stage that the child undertands that the last number word given in the forward count isn't only the largest and represents the last object counted, but also represents all the elements counted. This last developmental step is, in our opinion, very interesting, but our data do not allow us to specify how it is acquired. Our developmental sequence, while differing substantially from that of Gelman and Gallistel (1978), shares much in common with that of Fuson (1988), although we believe our findings allow us to propose a sequence better defined and more comprehensive.

Finally, although many authors (Fuson, 1988; Fuson et al., 1985b; Ginsburg & Russell, 1981; Wilkinson, 1984) claim that the size of the sets (from 2 to 19 approximately) does not have an effect on the response to the "how many" rule, our data show that this factor can be relevant to the cardinality response, as appears clearly in the counting backwards test. This is most probably due to the fact that the "How many" rule is related to the counting sequence, while cardinality is related to the set of objects as well.

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